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INERTIAL SURVEY ADJUSTMENT

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1. ABSTRACT

The paper discusses primarily a least-squares position adjustment for single or multiple (area coverage) inertial traverses. The adjustment technique is developed and presented in detail for a local-level inertial system and summarily generalized for a space-stable inertial system. Application of the method to other inertially derived geodetic values is discussed.

2. INTRODUCTION

Primary system errors affecting local-level inertial traversing for position include level accelerometer scale factor errors, initial platform azimuth error following alignment, platform azimuth drift during the mission, and level accelerometer non-orthogonality.

These errors combine to produce position errors dependent on the course taken while traversing. For example, east-west headings while traversing elicit longitude errors predominately caused by east axis scale factor error. At this same heading, latitude errors would be minimally contributed by north scale factor error and largely realized due to the other error parameters. While travelling north-south, north scale factor error is predominant for latitude error and the applicable error parameters (excluding east scale factor error) result in the preponderence of longitude error. The next section presents an error model based on this condition.

3. THE ERROR MODEL

In the following discussion level accelerometer non-orthogonality error has not been considered since this error parameter is highly correlated with the remaining attitude errors of more significance. Additionally, this error is a constant for surveys throughout an area and presurvey dynamic calibration techniques can be employed to reduce its contribution to position errors.

Regression equations including the remaining error terms are formulated for each point surveyed on a mission:

$$\Delta\lambda_i \mathrm{raw} - x(1) D_i \sin(A_i - x(3) - x(4) t_i) = 0 \quad (1)$$

$$\Delta\phi_i \mathrm{raw} - x(2) D_i \cos(A_i - x(3) - x(4) t_i) = 0 \quad (2)$$

In the equations $\Delta\lambda_i$ and $\Delta\phi_i$ are observed "raw" longitude and latitude changes respectively from the initial traverse station. The east and north scale factor errors are indicated by the x(1) and x(2) parameters respectively. D_i, A_i, and t_i are distance, azimuth, and time respectively to the surveyed point--calculated by employing mean forward-reverse "smoothed" coordinates. The initial platform azimuth error is reflected in the x(3) parameter and the platform azimuth drift is included in the x(4) term.

The subsequent set of equations may be solved by the method of least-squares for the parameters explaining the variation in the raw values along the traverse. With calculated parameters, the raw observations may be corrected with the equations:

$$\Delta\lambda_i = \sin(A_i) \Delta\lambda_i \mathrm{raw} / x(1) \sin(A_i - x(3) - x(4) t_i) \quad (3)$$

$$\Delta\phi_i = \cos(A_i) \Delta\phi_i \mathrm{raw} / x(2) \cos(A_i - x(3) - x(4) t_i) \quad (4)$$

These correction equations are seen to be unstable for azimuths near the cardinal directions, however, and should be avoided. Differentiation of these equations with respect to azimuth and application of typical error parameters reveals the azimuth instability range for medium length traverses of about 40km. When correcting raw $\Delta\lambda$'s and $\Delta\phi$'s within angular ranges of 10^{o} of cardinal east-west and north-south, respectively, the substitute equations below should be employed.

$$\Delta\lambda_i=(x(3)+x(4)t_i)(D_i\cos A_i)-(x(1)-1)(D_i\sin A_i)+\Delta\lambda_i raw \tag{5}$$

$$\Delta\phi_i=-(x(3)+x(4)t_i)(D_i\sin A_i)-(x(2)-1)(D_i\cos A_i)+\Delta\phi_i raw \tag{6}$$

The above correction equations are valid at any azimuth and are preferred.

On test traverses employing known positions for the D_i and A_i calculation, this approach has returned the known geodetic values to approximately 0.15m rms indicating the adequacy of the overall model. Unfortunately, smoothed coordinate derived azimuths are not relatively accurate enough to define parameters which result after application of correction equations in significantly improved smooth values on a single traverse basis.

The method has produced more favorable results, however, with application to multiple traverses for area coverage. In this case, only traverse crossing junction points and final closing known positions are used in the distance and azimuth calculations. The following section discusses these results obtained utilizing area coverage ground vehicle traversing at White Sands Missile Range (WSMR) in 1980.

4. ADJUSTMENT OF MULTIPLE TRAVERSES

Area coverage by inertial traversing with the Rapid Geodetic Survey System (RGSS) involved 11 different crossing traverse routes [1]. Each traverse was surveyed forward and reverse following system alignment at each end of the courses. At all traverse junction points, system smoothed values were meaned to provide best estimates of position for the distance and azimuth calculations for use in the regression equations. Using only these junction stations and known traverse end-point coordinates, 346 condition equations were written carrying 48 and 27 parameters for different test cases. The first adjustment carried four scale parameters, one each for east, south, west, and north travel along traverses; and, initial platform azimuth error and azimuth drift for each of the 22 missions totaling 48 parameters. The four scale parameters were carried to account for possible quantization differences identified by engineers when traveling in the plus and minus directions for each level axis. The 27 parameter test case included the four scale factor errors, 22 initial platform azimuth errors--one for each mission, and a single platform azimuth drift parameter for all missions. Apriori parameter sigmas were set at 0.02 percent for the scale factors initialized at one, 20 arcseconds for the platform azimuth errors estimated at zero, and 0.002 degree per hour for platform azimuth drift(s) initialized at zero.

Each of the above test cases was run with the mean of smoothed values at all junction points and with the known values utilized for distance and azimuth calculation at junctions. For the scale factors essentially the same values were estimated from all adjustments:

East travel scale factor	0.999999
South travel scale factor	1.000053
West travel scale factor	1.000061
North travel scale factor	1.000010

Three of these computed scale factors are realistic based on dynamic calibration experience at WSMR. The west scale factor is suspect in that it is possibly too large. Other scale errors were realized on 15km and 10km dynamic calibration baselines at WSMR.

Table 1 summarizes pertinent statistics for the platform azimuth error and drift parameter for each adjustment performed. These are external statistics. Internal (adjustment) statistics were quite low indicating consistency in the model. The aposteriori reference variance ($\hat{\sigma}_0^2$) is also shown in the table.

	48 Par sm jct	27 Par sm jct	48 Par kwn jct	27 Par kwn jct
	Platform Azimuth Error (arc-second)			
n	22	22	22	22
$\bar{x}$	-20.00	-21.3	-22.0	-21.4
σ	19.9	24.8	22.0	24.8
rms	28.0	32.3	29.2	32.3
max	+23.0	+35.5	+30.3	+35.6
min	-56.9	-74.4	-57.3	-74.2
σ_0^2	0.0047	0.0050	0.0047	0.0050
	Platform Azimuth Drift Error (degree/hour)			
n	22	1	22	1
$\bar{x}$	+0.0010	+0.0013	+0.0009	+0.0013
σ	0.0015	N/A	0.0019	N/A
rms	0.0018	N/A	0.0018	N/A
max	+0.0033	N/A	+0.0036	N/A
min	-0.0043	N/A	-0.0050	N/A

TABLE 1 - AZIMUTH ERROR STATISTICS

In the table above n- indicates the number of parameters estimated and $\bar{x}$ is the overall mean of all parameters of the same kind within an adjustment. The σ and rms are external standard deviation and root-mean-square estimates respectively. The max and min values show the highest and lowest parameter estimates respectively among all traverses. The σ_0^2 is the a posteriori reference variance estimate following adjustment. The a priori value was one.

Following parameter estimation the correction equations are employed to correct the raw coordinates utilizing the applicable parameters for the various traverses. Table 2 provides a summary of statistics as a result of this process. Adjustment input coordinate accuracies are given along with the accuracy of returned values at junction points and at stations intermediate to junctions not utilized for parameter estimation. A weighted and simple mean determination is provided for all interior points.

	48 Par sm jct	27 Par sm jct	48 Par kwn jct	27 Par kwn jct
	Junction Coordinates Input to Adjustment			
n	32	32	32	32
ϕ rms	0.20	0.20	0	0
λ rms	0.24	0.24	0	0
ϕ max	0.43	0.43	0	0
λ max	-0.63	-0.63	0	0
	Corrected Junction Coordinates (weighted mean)			
n	32	32	32	32
ϕ rms	0.18	0.21	0.14	0.19
λ rms	0.23	0.24	0.20	0.23
ϕ max	-0.37	0.40	-0.30	0.38
λ max	-0.68	-0.66	-0.54	-0.63
	Corrected Intermediate Coordinates (weighted mean)			
n	25	25	25	25
ϕ rms	0.21	0.27	0.21	0.26
λ rms	0.22	0.27	0.22	0.28
ϕ max	-0.52	0.53	-0.50	0.51
λ max	-0.46	0.75	-0.47	0.76
	Corrected Junction Coordinates (simple mean)			
n	32	32	32	32
ϕ rms	0.25	0.26	0.20	0.25
λ rms	0.26	0.26	0.24	0.25
ϕ max	0.51	0.51	-0.49	0.51
λ max	-0.88	-0.88	-0.84	-0.86
	Corrected Intermediate Coordinates (simple mean)			
n	25	25	25	25
ϕ rms	0.26	0.31	0.25	0.30
λ rms	0.26	0.28	0.26	0.29
ϕ max	0.71	0.73	0.72	0.73
λ max	-0.49	-0.56	-0.47	-0.54

TABLE 2 - ADJUSTMENT CORRECTED COORDINATES

In the table above n- is the number of stations statistically evaluated. The ϕ rms and λ rms are latitude and longitude root-mean-square values respectively. The ϕ max and λ max values pertain to the largest error after adjustment/correction in latitude and longitude respectively.

In the weighted mean categories for all adjustments rms statistics are slightly reduced along with maximum errors. The weight associated with parameter corrected raw values for these categories was $1/t_i$, the t_i being the time to the point on the traverse from the initial station of the mission. This weighting system was used after it was noticed that the model frequently revealed lack of fit with increasing time/distance along traverses. This was the case despite the fact that the final pair of raw observed coordinates went into the adjustment with a relative weight ten times greater than intermediate stations which were equally weighted. These weights were calculated based on rms statistics given in [1]. The model inadequacy is believed to be related to azimuth and distance computation to be investigated. Because of this condition, the weighting scheme proved effective for these adjustments but may not be required if the model deficiency can be eliminated.

Another feature in Table 2 is the slightly better results with the 48 parameter model over the 27 parameter model. The latter model enforces significantly more coupling throughout the system but no advantage is realized since the drift parameter displays significant variability from mission to mission. It is important to keep in mind that the inertial system was premission calibrated (aligned) 22 times for these survey missions apparently contributing to the drift variability. The manufacturer has suggested that possibly the drift parameter could be stabilized by leaving the system on for several successive survey missions enabling accountability for only one drift parameter for the affected traverses.

Table 2 further illustrates that the 48 parameter (known junction input) adjustment in this test case has a 1-sigma noise level of about 0.17m for each coordinate returning rms ϕ and λ values of 0.14m and 0.20m respectively. For the 48 parameter (smoothed junction input) adjustment, approximately a 5 percent improvement in terms of rms is seen in coordinate values returned at the junctions. This is a modest improvement from simple mean values for junction points input for the least-squares regression parameter estimation. Network intermediate points, not used for parameter estimation but only corrected, were reduced from a simple double-pass mean rms level of 0.30m to 0.22m--approximately a 25 percent improvement. It is relevant to note that the single pass level statistics were approximately 0.42m.

5. CONCLUSION

This adjustment technique may be adapted for determination of other geodetic values and to inertial surveying with a space-stable mechanized system if adequate models can be identified. Error parameters applicable for this type of system would be initial attitude errors accommodated in a three by three direction cosine matrix relating the inertial frame to the local geodetic frame, additive drift rates for each axis about the geodetic frame, scale factor parameters, and non-orthogonality parameters which potentially may be eliminated by factory calibration or compensation. The number of parameters is greater with such a mechanization but more observations can be included since height changes would also be eimployed as observables; direction and distance would be in three dimensions.

Application of this adjustment/correction approach in general requires the use of the regression technique for parameter estimation and the availability of correction equations. With a local-level mechanization such as RGSS, a deflection of the vertical adjustment model could be formulated with the raw deflection change components equal to functions of time, level gyro drift about the north and east axes, and accelerometer bias change. Both drift and bias parameters are at least required for each axis to account for excessive non-linear trends. A schuler term should also be considered (a schuler type effect was apparent in some raw error plots illustrating positioning errors). Correction equations would have to employ the error parameters in time functions. Further investigations are required to determine the feasibility and details involved for application of this approach for deflection determination, gravity anomaly and height estimation.

When this approach can be successfully formulated and applied, its advantage is that of simple least-squares regression entertaining great redundancy with a relatively low number of parameters. With adequate parameter definition, modeling, and correction algorithms, stochastic information on computed

geodetic values is then made available by employing simple error propagation techniques.

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